

# AdS/CFT correspondence in a Friedmann-Lemaitre-Robertson-Walker brane

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According to the AdS/CFT correspondence conjecture, the Randall-Sundrum infinite braneworld is equivalent to four dimensional Einstein gravity with  $\mathcal{N} = 4$  super Yang-Mills fields at low energies. Here we derive a four dimensional effective equation of motion for tensor-type perturbations in two different pictures, and demonstrate their equivalence.

Braneworld scenarios proposed by Randall and Sundrum[1, 2] attracted much attention. Especially the second model[2] (RS II) has been investigated a lot as a model which realizes a new scheme of compactifying an extra dimension[3, 4]. In this model bulk dynamics is governed by the five dimensional Einstein equations with a negative cosmological constant, and ordinary matter fields are confined to a four dimensional brane located at a boundary of the bulk with  $Z_2$ -symmetry. The simplest unperturbed background is given by a five dimensional AdS bulk with a cosmological constant,

$$ds^2 = -\frac{\ell^2}{z^2} (-dt^2 + \delta_{ij} dx^i dx^j + dz^2),$$

where  $\ell$  is the curvature length of the AdS space. Latin indices are used for 3-dimensional spatial coordinates and are raised and lowered by using the Kronecker delta  $\delta_{ij}$ . In the original RS II model [2], a Minkowski brane placed at a fixed value of  $z$  was considered. This configuration becomes a solution by tuning the tension of the brane as  $\sigma = 3/4\pi G\ell^2$ , where  $G$  is Newton's constant. Soon the model was extended so as to realize a general expansion law of the universe on the brane [5, 6, 7, 8, 9]. In fact, once we introduce matter fields or detuned brane tension, the brane in general starts to move. As an easiest example, a moving brane in  $z$ -direction represents a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe. The scale factor on the brane is given by  $a = \ell/z(t)$ , and the Hubble parameter  $H$  is related to the brane motion as  $H = -\dot{z}/\ell\sqrt{1-\dot{z}^2}$ , where dot represents differentiation with respect to  $t$ . From the junction condition on the brane, a modified Friedmann equation

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \left( \rho + \frac{\rho^2}{2\sigma} \right) \quad (1)$$

follows, where  $\mathcal{H} \equiv aH$  and  $\rho$  is the total energy density of matter fields localized on the brane.

In the RS II model the relative correction to Newton's law at a distance  $r$  is suppressed by a factor  $\ell^2/r^2$ [10, 11]. Experiments of gravitational forces constrain  $\ell$  to be shorter than 0.1mm or so. The fundamental mass scale of five dimensional gravity,  $m_5 \equiv G_5^{-1/3}$ , is related to the four dimensional Planck mass  $m_{pl} \equiv 1/\sqrt{G}$  by  $m_5^3 = m_{pl}^2/\ell$ . Hence  $m_5$  must be larger than  $10^8 \text{GeV}$ , which is far beyond the energy scale that collider experiments can

reach. Therefore the most stringent constraint on models of this type is expected to be brought by examining the history of the early universe or by testing modification of gravitational forces.

Aiming at providing constraints from the cosmic microwave background radiation, many works on cosmological perturbations in RS II model have been done[12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. However, we have not clearly understood yet how to qualitatively estimate the leading order correction to the predictions of the standard cosmology. First of all, once we take into account the effect of higher dimensions, solving perturbation equations for given initial conditions is difficult even at the classical level since in general at least two dimensional field equation must be solved. Moreover, it is not very clear which initial conditions are appropriate in principle in the braneworld setup. One exceptional case is creation of a braneworld from nothing[27]. In this case the standard scheme for specifying boundary conditions for the wavefunction of the universe (no boundary or tunneling conditions) will work. Even if we restrict our consideration to such a well-posed setup, it is still a very tough problem to sum up the fluctuations from all independent Kaluza-Klein modes[20, 22]. Unless a de Sitter brane is concerned, decomposition of independent modes is quite non-trivial. Even if we succeed in solving all independent mode functions, there is a problem of ultra-violet divergence when we evaluate the value of a bulk field exactly on the brane[28, 29]. This problem has not been taken seriously in literature since the divergence is logarithmic for the expectation value of a squared bulk field. However, if we compute quantities which contains more differentiations of a field, divergence will become severer.

On the other hand, non-linear corrections to gravity on a Minkowski brane were also investigated[30, 31, 32]. In all cases examined so far, corrections are always suppressed by a factor  $\ell^2/r^2$ . Namely, any non-linear effects do not make it easier to discriminate braneworld models of the RS II type from the standard. However, we proposed a conjecture that there is no stationary large black hole solution in the RS II model[33, 34]. The conjecture indicates that a black hole localized on the brane evaporate even at the classical level. Some numerical works to construct a large black hole solution were done recently, but only small black holes were found[35, 36]. (See also [37, 38, 39].) This fact might be a supporting evidence for the conjecture, although different interpretation is also possible. If the conjecture is correct, the life time of a black hole is estimated as  $\tau \approx (M/M_\odot)^3 (1\text{mm}/\ell)^2 \times 1$

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year, where  $M$  and  $M_\odot$  are masses of a black hole and the sun, respectively. The above conjecture is based on the AdS/CFT correspondence[40, 41]. The AdS/CFT correspondence conjecture asserts that the effective action of  $\mathcal{N} = 4$  super Yang-Mills fields evaluated on the metric induced on the boundary is given by

$$W_{CFT} = S_{EH} + S_{GH} - S_1 - S_2 - S_3, \quad (2)$$

where  $S_{EH} = -\frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( {}^{(5)}R + \frac{12}{\ell^2} \right)$ ,  $S_{GH} = -\frac{1}{8\pi G_5} \int d^4x \sqrt{-{}^{(4)}g} K$ ,  $S_1 = -(3/8\pi G_5 \ell) \int d^4x \sqrt{-{}^{(4)}g}$ ,  $S_2 = -(\ell/32\pi G_5) \int d^4x \sqrt{-{}^{(4)}g} {}^{(4)}R$  and  $S_3 = -(\ell^3/64\pi G_5) \ln(z_0/z) \int d^4x \sqrt{-{}^{(4)}g} \left( {}^{(4)}R_{\mu\nu} {}^{(4)}R^{\mu\nu} - (1/3) {}^{(4)}R^2 \right)$ .  $z_0$  determines the renormalization scale.  $K$  is the trace of the extrinsic curvature of the boundary. Left hand side is the action for five dimensional gravity theory. The counter terms  $S_1$ ,  $S_2$  and  $S_3$  are necessary to cancel manifest dependencies on the boundary location. Using this equality, the action of the RS II model is rewritten as

$$\begin{aligned} S_{RS} &= 2(S_{EH} + S_{GH}) - 2S_1 + S_{matt} \\ &= 2S_2 + 2(W_{CFT} + S_3) + S_{matt}. \end{aligned} \quad (3)$$

We notice that  $2S_2$  is the ordinary four dimensional Einstein-Hilbert action, while  $W_{CFT} + S_3$  is the effective action for a conformal field theory (CFT) with an ultra-violet cutoff. The above formula indicates equivalence between the RS II model and four dimensional Einstein gravity with CFT. Here we note that the leading order corrections due to the bulk effect at the classical level in the five dimensional RS picture come from one-loop quantum effects of CFT in the four dimensional picture. So far direct and satisfactory confirmations of the equivalence between these two pictures are limited to the following two cases. One is the case of linear perturbations from Minkowski brane[42], and the other is the homogeneous cosmology[43]. Equivalence is satisfied up to  $O(\ell^2)$ .

The main purpose of this letter is to add another example for the equivalence between RS II model and four dimensional Einstein gravity with CFT by considering tensor-type perturbations on a FLRW brane. We derive the leading order corrections at low energies in both four and five dimensional pictures, and show that in fact they are identical. Although here in this short article we shall not pursue an application of our new results to understand cosmological perturbations in the braneworld, we expect that it might open up a new approach to this problem. In the four dimensional picture we just need to consider the backreaction due to vacuum polarizations of CFT, and there are only two graviton degrees of freedom for each three dimensional wave number. This fact may allow us to avoid the problems mentioned above in solving cosmological perturbations.

*Five dimensional RS picture:* First we discuss tensor-type perturbations on a FLRW brane in the five dimensional RS picture. The bulk metric is given by

$$ds^2 = \frac{\ell^2}{z^2} (-dt^2 + (\delta_{ij} + h_{ij}) dx^i dx^j + dz^2). \quad (4)$$

By using  $Y_k^{ij}$ , a transverse traceless tensor harmonics normalized as  $\int d^3x Y_{kij}(\mathbf{x}) Y_{k'ij}(\mathbf{x}) = \delta^3(\mathbf{k} - \mathbf{k}')$ , we expand perturbations as  $h_{ij} = Y_{kij} \Phi$ . In these coordinates the five dimensional perturbation equation reduces to

$$\left( -\partial_z^2 + \frac{3}{z} \partial_z + \partial_t^2 + k^2 \right) \Phi = 0. \quad (5)$$

The general solution to this equation is given by

$$\begin{aligned} \Phi &= \int d\omega \tilde{\Psi}(\omega) e^{-i\omega t} 2(pz)^2 K_2(pz) \\ &= \int d\omega \tilde{\Psi}(\omega) e^{-i\omega t} \\ &\quad \times \left[ 1 - \frac{(pz)^2}{4} + \frac{(pz)^4}{16} (b - \ln(pz)) + \dots \right], \end{aligned} \quad (6)$$

where  $p^2 = -(\omega + i\epsilon)^2 + k^2$ ,  $b = \frac{1}{2}(\frac{3}{2} - 2\gamma) + \ln 2$ , and  $\gamma$  is Euler's constant. Here we have chosen the branch cut of the modified Bessel function  $K_2$  so that there is no incoming wave from past null infinity in the bulk. What we have to do is to choose  $\tilde{\Psi}$  such that  $\Phi$  satisfies the perturbed junction condition  $n^\rho \partial_\rho \Phi = 0$ , where  $n^\rho$  is the unit normal to the brane. Notice that the conformal time of the induced metric on the brane  $\eta$  is related to the bulk conformal time  $t$  by  $d\eta = \sqrt{1 - (dz(t)/dt)^2} dt$ . The derivative along the direction parallel to the brane is given by  $\partial_\eta = \sqrt{1 + (H\ell)^2} \partial_t - H\ell \partial_z$ . Thus the unit normal is given by  $n^\mu \partial_\mu = a^{-1}(-H\ell \partial_t + \sqrt{1 + H^2 \ell^2} \partial_z)$ .

Here we use an alternative approach. In Ref. [45] effective Einstein equations

$${}^{(4)}G^\mu{}_\nu - 8\phi G_4 T^\mu{}_\nu = (8\pi G_5)^2 \pi^\mu{}_\nu - E^\mu{}_\nu, \quad (7)$$

were derived, where  $\pi^\mu{}_\nu$  is a tensor quadratic in the energy momentum tensor  $T^\mu{}_\nu$  and  $E^\mu{}_\nu$  is a projected Weyl tensor defined by  $n^\rho n^\sigma C^\mu{}_{\rho\nu\sigma}$ . From Eq. (7) the effective four dimensional equation for tensor perturbations is given by

$$(\partial_\eta^2 + 2H\partial_\eta + k^2) \phi = -2E, \quad (8)$$

with  $\phi \equiv \Phi|_{z=z(t)}$ , and the correction due to  $E_{\mu\nu}$  is explicitly given as

$$\begin{aligned} -2E &= \left\{ (H\ell)^2 (\partial_t^2 + \partial_z^2) - 2H\ell \sqrt{1 + (H\ell)^2} \partial_t \partial_z \right. \\ &\quad \left. + \left( \partial_z^2 - \frac{1}{z} \partial_z \right) \right\} \Phi \Big|_{z=z(t)}, \end{aligned} \quad (9)$$

To Solve Eq. (5) supplemented with Eq. (8) is equivalent to solve the same equation with the perturbed junction condition,  $n^\rho \partial_\rho \Phi = 0$ . To see deviation from the four dimensional Einstein gravity, the former is more convenient.

At low energies (when  $H^2 \ell^2 \ll 1$ ) further reduction is possible in an approximate sense like

$$\begin{aligned} \partial_t^2 \Phi &\approx \partial_\eta^2 \Phi, \\ \partial_z^2 \Phi &\approx -\frac{1}{2} \int d\omega \tilde{\Psi} e^{-i\omega \eta} p^2 \approx -\frac{1}{2} (\partial_\eta^2 + k^2) \Phi, \end{aligned}$$

$$\partial_t \partial_z \Phi \approx -\frac{\ell}{2a} \partial_\eta (\partial_\eta^2 + k^2) \Phi. \quad (10)$$

Here we have neglected higher order corrections of  $O(\ell^4)$ . Only the last term  $(\partial_z^2 - \frac{1}{z} \partial_z) \Phi$  does not allow such a simple reduction because the third term in the expansion of  $K_2(pz)$ , which is not a polynomial in  $p^2$ , contributes to this term. This term does not have explicit  $\ell^2$  suppression at this level. However, first two terms in the expansion of  $K_2(pz)$  vanish for this combination of differentiation. As a result, a factor of  $z^2 = \ell^2/a^2$  arises. We finally obtain

$$\begin{aligned} & (\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2) \phi \\ & \approx \left[ \frac{(H\ell)^2}{2} (\partial_\eta^2 - k^2) + \frac{H\ell^2}{a} \partial_\eta (\partial_\eta^2 + k^2) \right] \phi \\ & + \frac{\ell^2}{2a^2} \int d\omega \tilde{\phi} e^{-i\omega\eta} p^4 \left( b - \frac{3}{4} - \ln \left[ \frac{p\ell}{a} \right] \right). \quad (11) \end{aligned}$$

A similar equation was derived for scalar-type perturbations in Ref. [17]. All the corrections are suppressed by  $\ell^2$  or  $\ell^2 \ln \ell$ . The first term on the right hand side can be rewritten by using the lower order equation as

$$\frac{\ell^2}{a^2} [(3\mathcal{H}^3 - 2\mathcal{H}\mathcal{H}') \partial_\eta + k^2 \mathcal{H}^2] \phi.$$

*Four dimensional CFT picture:* Effective equations of motion for tensor perturbations in CFT picture have already been obtained in Ref. [46]. Here we give a brief derivation of it. The equation for the tensor-type perturbation is given in the form of

$$\partial_\eta a^2 \partial_\eta \phi + a^2 k^2 \phi - 16\pi G a^2 \tau = 0, \quad (12)$$

where the perturbation variable  $\phi$  is defined as before, and  $\tau(\eta) \equiv \int d^3x T_{ij}^{(CFT)}(\eta, \mathbf{x}) Y_k^{ij}(\mathbf{x})$  is the contribution from the effective energy momentum tensor of CFT. In order to evaluate the energy momentum tensor due to vacuum polarization of CFT, we can make use of the fact that the metric of flat perturbed FLRW universe is related via conformal transformation to Minkowski space-time with the corresponding perturbations as

$$ds_{(1)}^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) ds_{(0)}^2, \quad (13)$$

where  $ds_{(0)}^2 = g_{\mu\nu}^{(0)} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu$ . We consider one parameter family of conformally related metrics  $g_{\mu\nu}^{(\theta)} = a^{2\theta} g_{\mu\nu}^{(0)}$  connecting  $g_{\mu\nu} \equiv g_{\mu\nu}^{(1)}$  with  $g_{\mu\nu}^{(0)}$ . The action for CFT is invariant under conformal transformation except for  $T_{\mu\nu}^{(A)}$ , the contribution from the conformal anomaly. Hence, the energy momentum tensor of CFT excluding the anomaly contribution transforms in a trivial manner under a conformal transformation. Thus we have

$$T_{\mu\nu}^{(CFT)} = a^{-2} T_{\mu\nu}^{(0)} + T_{\mu\nu}^{(A)}, \quad (14)$$

where  $T_{\mu\nu}^{(0)}$  is the CFT energy momentum tensor evaluated on the metric  $g_{\mu\nu}^{(0)}$ . Correspondingly, we divide  $\tau$  into two pieces,  $\tau^{(0)}$  and  $\tau^{(A)}$ .

From the comparison of Eqs. (10), (12) and (13) in Ref.[42],  $\tau^{(0)}$  is found to be given by

$$-16\pi G a^2 \tau^{(0)} = 2 \int d\omega e^{-i\omega\eta} \Pi_2(p^2) p^4 \tilde{\phi}(\omega), \quad (15)$$

with

$$\Pi_2(p^2) = \frac{\ell^2}{8} \ln \frac{p^2}{\mu^2}, \quad (16)$$

where we used the properties of tensor-type perturbations,  $h^{0\alpha} = 0$ ,  $h_j^j = 0$ , and  $h^{ij} k_j = 0$ . Constant part in  $\Pi_2$  was absorbed by the renormalization scale  $\mu$ . Here  $\tilde{\phi}(\omega) \equiv (1/2\pi) \int d\eta e^{i\omega\eta} \phi(\eta)$  is the Fourier transform of  $\phi(\eta)$ .

Next we consider the anomaly contribution  $\tau^{(A)}$ . The anomaly contribution to the effective action,  $S^{(A)}$ , is given by an integral of the Seeley-deWitt coefficient as[41, 44]

$$S^{(A)} = -\frac{1}{16\pi^2} \int_0^1 d\theta \int d^4x a_2^\theta \ln a, \quad (17)$$

with  $a_2^\theta = \frac{N^2}{6} \sqrt{-g^{(\theta)}} (3R_{\mu\nu} R^{\mu\nu} - R^2) [g^{(\theta)}]$ , and  $N^2 = \pi\ell^2/G$  is the number of degrees of freedom of CFT. The anomaly contribution to the energy momentum tensor is obtained by taking variation of  $S^{(A)}$  as  $T_{\mu\nu}^{(A)} = -\frac{1}{2} (\delta S^{(A)} / \delta g^{\mu\nu})$ . Expanding the action up to second order in  $h_{\mu\nu}$ , we obtain

$$\begin{aligned} S^{(A)} &= \frac{\ell^2}{64\pi G} \int d\eta \left( 2\mathcal{H}^4 \right. \\ &\quad \left. + \sum_k \phi \left[ \ln a (\partial_\eta^2 + k^2)^2 + \mathcal{O} \right] \phi \right), \quad (18) \end{aligned}$$

where

$$\begin{aligned} \mathcal{O} &= 2\mathcal{H}\partial_\eta^3 + (\mathcal{H}' + \mathcal{H}^2) \partial_\eta^2 + ((\mathcal{H}^2)' + \mathcal{H}k^2) \partial_\eta \\ &\quad + k^2 (\mathcal{H}' - \mathcal{H}^2) + 4\mathcal{H}'\mathcal{H}^2 - \mathcal{H}^4. \quad (19) \end{aligned}$$

The first term in the round brackets in Eq. (18) gives a correction to the background Friedmann equation. Variation of the action with setting  $\phi = 0$  leads to

$$aa'' = \frac{4\pi G}{3} a^4 (\rho - 3P) + \frac{\ell^2}{2} \mathcal{H}^2 \mathcal{H}'. \quad (20)$$

Using the continuity equation  $\rho' = -3\mathcal{H}(\rho + P)$ , we can integrate Eq. (20) once to obtain

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho + \frac{\ell^2}{4a^2} \mathcal{H}^4 + \frac{C}{a^2}. \quad (21)$$

Here  $C$  arises as an integration constant. This term represents the so-called dark radiation. It is easy to verify the equivalence between Eqs. (1) and (20) up to  $O(\ell^2)$  when  $C = 0$ .

The  $\phi$ -dependent part of the anomaly contribution is given by

$$16\pi G a^2 \delta \tau^{(A)} = 16\pi G \frac{\delta S^{(A)}}{\delta \phi}$$

$$= \frac{\ell^2}{2} \left[ \ln a (\partial_\eta^2 + k^2)^2 + \mathcal{O} \right] \phi. \quad (22)$$

Combining two contributions  $\tau^{(0)}$  and  $\delta\tau^{(A)}$ , using the lower order equation, we can write down the modified equation of motion for  $\phi$  as

$$\begin{aligned} & (\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 + (4\mathcal{H}' + 2\mathcal{H}^2) + 16\pi G a^2 P) \phi \\ & \approx \frac{\ell^2}{a^2} \left[ (3\mathcal{H}^3 - 2\mathcal{H}\mathcal{H}') \partial_\eta + k^2\mathcal{H}^2 + 2\mathcal{H}^2\mathcal{H}' - \frac{1}{2}\mathcal{H}^4 \right] \phi \\ & - \frac{\ell^2}{2a^2} \int d^4p \tilde{\phi} e^{-i\omega\eta} p^4 \ln(p/a\mu). \end{aligned} \quad (23)$$

Using the background equation, we can rewrite the above equation as

$$\begin{aligned} & (\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2) \phi \\ & \approx \frac{\ell^2}{a^2} \left[ (3\mathcal{H}^3 - 2\mathcal{H}\mathcal{H}') \partial_\eta + k^2\mathcal{H}^2 + \frac{2C}{a^2} \right] \phi \\ & - \frac{\ell^2}{2a^2} \int d^4p \tilde{\phi} e^{-i\omega\eta} p^4 \ln\left(\frac{p}{a\mu}\right). \end{aligned} \quad (24)$$

Hence, we find that the above equation for tensor-type perturbations is identical to Eq. (11) obtained in the five dimensional RS II picture, as far as the dark radiation term, which we neglected in deriving Eq. (11), is set to zero.

In the cosmological context basically we have two non-dimensional quantities of  $\mathcal{O}(\ell^2)$ . One is  $\ell^2 H^2$  and the other is  $\ell^2 k^2/a^2$ . In this paper we developed a perturbative expansion scheme which is valid at low energies up to  $\mathcal{O}(\ell^2)$  since we are interested in the AdS/CFT correspondence. However, the method taken in the five dimensional RS II picture admit an extension to more general cases relaxing the constraint  $\ell H^2 \ll 1$ . This extension will be discussed in our forthcoming paper.

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